Benha University
Faculty of Engineering- Shoubra
Electrical Engineering Department
2^{nd} Year electrical power

Final Term Exam
Date: 11^{th} of June 2012
Mathematics 3(B) Code: EMP 272
Duration : 3 hours

- Answer all the following questions
- Illustrate your answers with sketches when necessary.
- The exam. Consists of one page

- No. of questions:4
- Total Mark: 80 Marks

1- Solve the system $x + 4y + z = 2$, $4x + y + z = 5$, $x + y + 4z = 3$ using:

i) An iterative method ii) Cholesky decomposition iii) Gauss Jordan method

20 Marks

2- i) **Find** y(1.1) using **modified Euler** method for the differential equation:

$$x` = x^2 - 2tx + y - 2t,\quad y` = y - x^2 - 2tx + 2t + 3,\ x(1) = 2,\ y(1) = 3\ ,\ h = 0.1$$

ii) $x` = -10(x-y),\quad y` = -xz + 28x - y,\ z` = xy - 8z/3,\ x(0) = 2,\ y(0) = -1,\ z(0) = 3,\ h = 0.05$

Solve the above system using **Picard** method and find x(0.1) using **Euler** method.

20 Marks

3-i) Consider the problem of determining the steady state heat distribution in a thin square metal plate with dimensions 0.5m by 0.5m. Two adjacent boundaries are held at $0^\circ c$ and the heat of the other boundaries increases linearly from $0^\circ c$ at one corner to $100^\circ c$ where the sides meet. The problem is expressed as $\mathbf{u_{xx} + u_{yy} = 10x}$. If the grid is divided into 5 equal parts, find u(x,y) such that k = 0.25. Solve the constructed linear system of equations using Gauss elimination method.

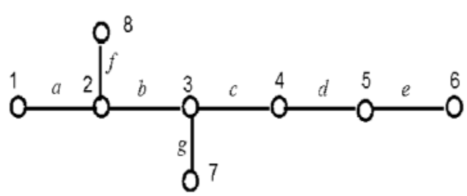
ii) Find the constants of the curve $\mathbf{y = a\cos x + b\ln x + c\,e^{x/10}}$ that fit **(1,3), (5,14), (19,101)**

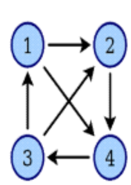
20 Marks

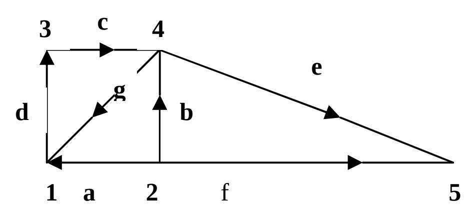
4-i) **Define** with an **example** for each of the following terms:

Simple Graph – Valency – Walk - Trail – Path - Complete Graph - Null Graphs - Bipartite Graphs - Tree Graph - Spanning Tree - Connected Graphs - Multi Graphs- Eulerian circuit – Eulerian path - Hamiltonian path -

ii) **Find** incidence and adjacency matrices for the following graphs







20 Marks

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Modified Euler states: $y_{i+1} = y_i + (h/2)[f(x_i, y_i) + f(x_{i+1}, y_i + hf(x_i, y_i))]$

Model answer

1-i) Using Gauss–Seidel:

Rarrange: $4x + y + z = 5$, $x + 4y + z = 2$, $x + y + 4z = 3$

$x^{(k+1)} = [5- y^{(k)} - z^{(k)}]/4$, $y^{(k+1)} = [2- x^{(k+1)} - z^{(k)}]/4$, $z^{(k+1)} = [3- x^{(k+1)} - y^{(k+1)}]/4$

Let $(x,y,z)^{(0)} = (0,0,0)$, therefore the 1st iteration will be: $x^{(1)} = [5- y^{(0)} - z^{(0)}]/4 = 1.25$,

$y^{(1)} = [2- x^{(1)} - z^{(0)}]/4 = 0.1875$, $z^{(1)} = [3- x^{(1)} - y^{(1)}]/4 = 0.3906$ and the 2nd iteration will be

$x^{(2)} = [5- y^{(1)} - z^{(1)}]/4 = 1.1055$, $y^{(2)} = [2- x^{(2)} - z^{(1)}]/4 = 0.126$, $z^{(2)} = [3- x^{(2)} - y^{(2)}]/4 = 0.4421$

1-ii)Using Cholesky method

$$\begin{pmatrix} 2 & 0 & 0 \\ 0.5 & 1.9365 & 0 \\ 0.5 & 0.3873 & 1.8974 \end{pmatrix}\begin{pmatrix} 2 & 0.5 & 0.5 \\ 0 & 1.9365 & 0.3873 \\ 0 & 0 & 1.8974 \end{pmatrix} = \begin{pmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 & 0 \\ 0.5 & 1.9365 & 0 \\ 0.5 & 0.3873 & 1.8974 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}, \text{ therefore} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2.5 \\ 0.3873 \\ 0.8433 \end{pmatrix}$$

$$\text{Also} \begin{pmatrix} 2 & 0.5 & 0.5 \\ 0 & 1.9365 & 0.3873 \\ 0 & 0 & 1.8974 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2.5 \\ 0.3873 \\ 0.8433 \end{pmatrix}, \text{ therefore} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1.1111 \\ 0.1111 \\ 0.4445 \end{pmatrix}$$

1-iii) Using Gauss Jordan method:

$$\left(\begin{array}{ccc|c} 4 & 1 & 1 & 5 \\ 1 & 4 & 1 & 2 \\ 1 & 1 & 4 & 3 \end{array}\right) \approx \left(\begin{array}{ccc|c} 4 & 1 & 1 & 5 \\ 0 & -15 & -3 & -3 \\ 0 & -3 & -15 & -7 \end{array}\right) \approx \left(\begin{array}{ccc|c} 60 & 0 & 12 & 72 \\ 0 & -15 & -3 & -3 \\ 0 & 0 & 72 & 32 \end{array}\right) \approx \left(\begin{array}{ccc|c} 360 & 0 & 0 & 400 \\ 0 & -360 & 0 & -40 \\ 0 & 0 & 72 & 32 \end{array}\right)$$

$$\text{Therefore} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10/9 \\ 1/9 \\ 4/9 \end{pmatrix}$$

2-i) $\mathbf{x` = x^2 - 2tx + y - 2t = f(x,y,t)}$, $\mathbf{y` = y - x^2 - 2tx + 2t + 3 = \varphi(x,y,t)}$, $x_0 = 2$, $y_0 = 3$, $t_0 = 1$

$$y_{i+1} = y_i + (h/2)[\varphi(t_i, x_i, y_i) + \varphi(t_{i+1}, x_i + hf(t_i, x_i, y_i), y_i + h\varphi(t_i, x_i, y_i))]$$

Put i = 0, therefore

$$y_1 = y_0 + (h/2)[\varphi(t_0, x_0, y_0) + \varphi(t_1, x_0 + hf(t_0, x_0, y_0), y_0 + h\varphi(t_0, x_0, y_0))] = 2.9585$$

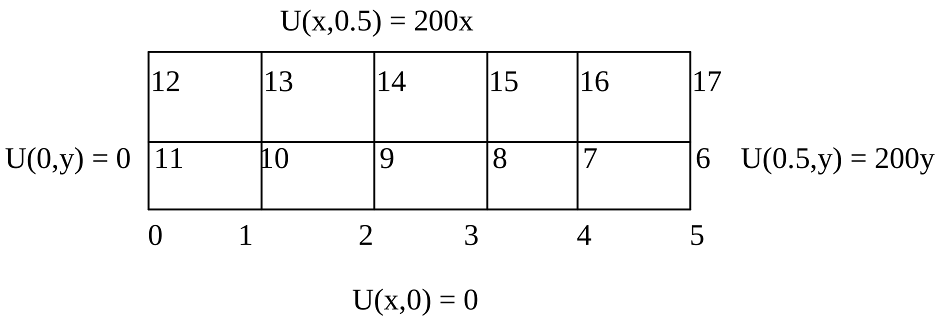
2-ii) $y_{n+1} = y_0 + \int_{t_0}^{t} (x_n z_n + 28x_n - y_n)\,dt$, $x_{n+1} = x_0 + \int_{t_0}^{t} -10(x_n - y_n)\,dt$,

$z_{n+1} = z_0 + \int_{t_0}^{t} (x_n y_n - 8z_n/3)\,dt$, $y_0 = -1$, $x_0 = 2$, $t_0 = 0$, $z_0 = 3$, thus $x_1 = x_0 + \int_{t_0}^{t} -10(x_0 - y_0)\,dt$,

$y_1 = y_0 + \int_{t_0}^{t} (x_0 z_0 + 28x_0 - y_0)\,dt$ and $z_1 = z_0 + \int_{t_0}^{t} (x_0 y_0 - 8z_0/3)\,dt$, therefore $x_1 = 2-30t$, $y_1=-1+51t$, $z_1 = 3-10t$. Similarly, $x_2=x_0+\int_{t_0}^{t} -10(x_1 - y_1)\,dt$, $y_2 = y_0 + \int_{t_0}^{t} (x_1 z_1 + 28x_1 - y_1)\,dt$ and $z_2= z_0 + \int_{t_0}^{t} (x_1 y_1 - 8z_1/3)\,dt$, therefore $x_2 = 2 - 30t + 405t^2$, $y_2 =-1+51t-(781/2)t^2 - 100t^3$, $z_2 = 3-10t +(238/3)t^2 - 510t^3$.

2nd : using Euler, $x_{n+1} = x_n + h\,[-10(x_n - y_n)]$, $y_{n+1} = y_n + h\,[-x_n z_n + 28 x_n - y_n]$, thus $x_1 =x_0 +h[-10(x_0 - y_0)] = 0.5= x(0.05)$, $y_1 = y_0 +h\,[-x_0 z_0 + 28 x_0 - y_0] =1.55 = y(0.05)$, therefore $x(0.1) = x_2 = x_1 +h[-10(x_1 - y_1)] = 1.025$

3-i)



$u_0 = u_1 = u_2 = u_3 = u_4 = u_5 = u_{11} = u_{12} =0$, $u_6 =50$, $u_{17} =100$, $u_{16} =80$, $u_{15} = 60$, $u_{14} =40$, $u_{13} =20$

The formula of Poisson equation is simplified to:

$$\mathbf{0.0625[\,U_{i+1,j} + U_{i-1,j}\,] + 0.01[\,U_{i,j+1} + U_{i,j-1}\,] - 0.00625\, x_i = 0.145U_{i,j}}$$

From which the following system of equations are constructed:

$0.0625\, U_8 - 0.145\, U_7 = -3.9225$, $0.0625[\,U_7 + U_9\,]- 0.145\, U_8 = - 0.59813$,

$0.0625[\,U_8 + U_{10}\,]- 0.145\, U_9 = - 0.39875$, $0.0625\, U_9 - 0.145\, U_{10} = -0.1994$,

3-ii) To get constants a,b,c, we have to use Least square method Such that

$$\sum_{i=1}^{3} y_i \cos(x_i) = a\sum_{i=1}^{3}[\cos(x_i)]^2 + b\sum_{i=1}^{3}[\cos(x_i)][\ln(x_i)] + c\sum_{i=1}^{3}[\cos(x_i)][e^{-x_i/10}]$$

$$\sum_{i=1}^{3} y_i [\ln(x_i)] = a\sum_{i=1}^{3}[\cos(x_i)][\ln(x_i)] + b\sum_{i=1}^{3}[\ln(x_i)]^2 + c\sum_{i=1}^{3}[\ln(x_i)][e^{-x_i/10}]$$

$$\sum_{i=1}^{3} y_i [e^{-x_i/10}] = a\sum_{i=1}^{3}[\cos(x_i)][e^{-x_i/10}] + b\sum_{i=1}^{3}[\ln(x_i)][e^{-x_i/10}] + c\sum_{i=1}^{3}[e^{-x_i/5}]$$

$\sum_{i=1}^{3}[\cos(x_i)]^2 = 1.3499$, $\sum_{i=1}^{3}[\cos(x_i)][\ln(x_i)] = 3.3677$, $\sum_{i=1}^{3}[\cos(x_i)][e^{-x_i/10}] = 7.6751$

$\sum_{i=1}^{3}[\ln(x_i)]^2 = 11.2597$, $\sum_{i=1}^{3}[\ln(x_i)][e^{-x_i/10}] = 22.3394$ and $\sum_{i=1}^{3}[e^{-x_i/5}] = 48.641$, from which we can get a, b, c

4-i) **Simple Graph**: A graph with no loops or multiple edges is called a simple graph
Valency:Is the degree of vertices
Walk: Pass through vertices and edges of the graph and may pass through repeated vertices and edges
Trail: If all the edges (but no necessarily all the vertices) of a walk are different, then the walk is called a trail (i.e. walk with no repeated edges)
Path: All edges and vertices of walk are different, then the trail is called path(i.e. trail with no repeated vertices) .
Complete Graphs: Is a graph in which every two distinct vertices are joined by exactly one edge
Null Graphs: graph containing no edges
Bipartite Graphs: Is a graph whose vertex-set can be split into two sets in such a way that each edge of the graph joins a vertex in first set to a vertex in second set.

Tree Graph: A tree is a connected graph which has no cycles.

Spanning Tree: If G is a connected graph, the spanning tree in G is a sub graph of G which includes every vertex of G and is also a tree.

Connected Graphs: A graph G is connected if there is a path in G between any given pair of vertices, otherwise it is disconnected

Multi Graphs: A multigraph or pseudograph is a graph which is permitted to have multiple edges
Eulerian circuit: Is a Eulerian trail which starts and ends on the same vertex
Eulerian path: Is a trail in a graph which visits every edge exactly once.

Hamiltonian path: Is a path in a graph G that passes through every vertex exactly once.

Incidence matrices:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} -1 & -1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & -1 \\ 0 & 1 & -1 & 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 1 \\ -1 & -1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

Adjacency matrices:

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$